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FORECASTING AND TIME SERIES MODEL TYPES OF 111

ECONOMIC TIME SERIES

by

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Forecasting, ARARMA time series models, time series memory, economic time series analysis.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) "Is it possible to put an end to the argument of what forecasting methods are better and under what circumstances?" Makridakis has organized a "forecasting competition" to which various forecasting experts would contribute forecasts of 111 economic and business time series which he has collected. This paper reports the results of our analysis of these series. An appendix describes the theory of univariate time series modeling and forecasting used in this study. The main text summarizes the diverse models which are encompassed by our approach, and which arise in the study of the 111 time series being forecasted.

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INTRODUCTION

"Forecasting and Time Series Model Types of 111 Economic Time Series"
is a chapter to be published in a book Major Time Series Methods and Their
Relative Accuracy by S. Makridakis, A. Andersen, R. Carbone, R. Fildes,
M. Hibon, R. Lewandowski, J. Newton, E. Parzen, and R. Winkler, Wiley: London,
1983. It reports in detail the forecasting procedure followed by Parzen and
Newton in their participation in the forecasting "competition" whose results
are reported in Makridakis, S., et al (1982) "The Accuracy of Extrapolation
(Time Series) Methods: Results of a Forecasting Competition," Journal of
Forecasting, 1, 111-153.

The joint paper did not explicitly draw any conclusions concerning
which methods performed best. Commentaries on the joint paper (to appear in
1983 in the Journal of Forecasting) seem to acknowledge the excellence of
the forecast errors obtained by Parzen and Newton. David J. Pack points out
the desirability of increasing the numeracy of the joint paper's Table 2(b),
which provides MAPE measures of how well each forecasting method performed
for the entire 111 series sample [reproduced in Pack's Exhibit 1]. Pack's
Exhibit 2 is the same table with methods ordered to the "average of
forecasting horizons 1-12" column, and all MAPE's divided by 13.4, the minimum
MAPE in the ordering column.

We reproduce Pack's Exhibits 1 and 2. Readers must draw their own
conclusions concerning the superiority of the forecasting methods used by
Parzen and Newton. Our contribution to the commentaries on the joint paper
is printed at the end of this report with the title "How to Learn from the JoF
Competition."

EXHIBIT 1. THE PAPER TABLE 2(A). AVERAGE MAPE ALL DATA(III)

EXHIBIT 2. PAPER TABLE 2(B) ORDERED TO HORIZON 1-12 AVERAGE AND RATIOED TO 13.9

FORECASTING AND TIME SERIES MODEL TYPES OF 111
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H. JOSEPH NEWTON
and
EMANUEL PARZEN

1. Introduction

"Is it possible to put an end to the argument of what forecasting methods are better and under what circumstances?", is the question raised by Professor Spyros Makridakis in several stimulating papers (1976), (1978), (1979). He has organized a "forecasting competition" to which various forecasting experts would contribute forecasts of 111 economic and business time series which he has collected. This paper reports the results of our analysis of these series, based on the general approach to time series modeling, spectral analysis, and forecasting developed by Parzen, with the collaboration of Newton.

An appendix describes the theory of univariate time series modeling and forecasting used in this study. The main text summarizes the diverse models which are encompassed by our approach, and which arise in the study of the 111 time series being forecasted.

The methods of time series modeling and forecasting applied in this paper can be applied automatically but they are not rote formulas, since they are based on a flexible philosophy which provides several models for consideration and diverse diagnostics for qualitatively and quantitatively checking the fit of a model (see Parzen (1979), (1980), (1981)). The models considered are called ARARMA models because the model computed adaptively for a time series is based on sophisticated time series analysis of ARMA schemes (a short memory model) fitted to residuals of simple extrapolation (a long memory model obtained by parsimonious "best lag" non-stationary autoregression).

A consumer of time series forecasting and/or modeling methods must evaluate the value of a proposed procedure in the context of the actual time series with which he, or she, is concerned. Our approach aims to be applicable in all the diverse fields to which time series analysis is being applied.

A major problem of time series forecasting is whether long range forecasting and short range forecasting require different methods to obtain satisfactory forecasts. This paper describes iterated models which provide qualitative diagnostics as to the possibility of long range forecasts (by diagnosing whether the time series is long memory). Both long range and short range forecasts are provided by a model obtained by fitting a parsimonious non-stationary autoregression whose residuals $Y(t)$ are modeled by a

stationary autoregression.

The modeling procedure is both automatic and flexible. In particular, two model orders are determined for $\tilde{Y}(t)$ and we would recommend computing and comparing forecasts from both models.

This paper aims to illustrate the results one obtains by typical graphs, and to describe the time series model types that one should expect to encounter when dealing with many economic time series.

2. Iterated Models Approach to Time Series Analysis

The problem of forecasting future values of a time series from observations of its past values has an extensive literature which propose many different approaches. The approach adopted here aims to fit automatically to a time series sample not one but several models. The class of models considered is suitable for time series modeling, spectral analysis, and forecasting and for time series encountered by researchers in the physical sciences, engineering sciences, biological sciences, and medicine, as well as to the social sciences, economics, and management sciences.

A time series may be predictable for a long time in the future or only over a limited future. We say the former has "long memory" and the latter "short memory". A time series with long memory requires a "non-stationary" model with periodic, cycle, and trend components. A time series with short memory requires a "stationary" model which is a linear filter relating the time series to its innovations or random shocks. The linear filter is an AR, MA, or ARMA filter (autoregressive, moving average, or mixed autoregressive-moving average).

The model we fit to a time series $Y(.)$ is an iterated model

$$Y(t) \ -\square\!\!\rightarrow \ \tilde{Y}(t) \ -\square\!\!\rightarrow \ \epsilon(t)$$

If needed to transform a long memory series Y to a short memory series $\tilde{Y}$, $\tilde{Y}(t)$ is chosen to satisfy one of the three forms

$$\tilde{Y}(t) = Y(t) - \hat{\phi}(\hat{\tau}) \ Y(t-\hat{\tau}) \ , \qquad (1)$$

$$\tilde{Y}(t) = Y(t) - \hat{\phi}_1 \ Y(t-1) - \hat{\phi}_2 \ Y(t-2) \ . \qquad (2)$$

$$\tilde{Y}(t) = Y(t) - \hat{\phi}_1 \ Y(t-\tau -1) - \hat{\phi}_2 \ Y(t-\tau) \qquad (3)$$

Usually $\tilde{Y}(t)$ is short memory; then it is transformed to a white noise, or no memory, time series $\epsilon(t)$ by an approximating autoregressive scheme AR(m) whose order m is chosen by an order determining criterion (we use CAT, introduced by Parzen (1974),(1977)).

In the present study, $\tilde{Y}(t)$ was found to be always short memory. It is then modeled by a stationary autoregressive scheme. It is argued by Parzen that approximating AR schemes suffice for spectral analysis and forecasting. Only for model interpretation is it desirable to fit an ARMA scheme. In the present study not more than 15 percent of the time series could be regarded as requiring an ARMA scheme.

To determine the best lag $\hat{\tau}$, we use non-stationary autoregression; either fix a maximum lag M and choose $\hat{\tau}$ as the lag minimizing over all τ

$$\sum_{t=M+1}^{T} \{Y(t) - \phi(\tau) \ Y(t-\tau)\}^2$$

or choose $\hat{\tau}$ as the lag minimizing over all τ

$$\sum_{t=\tau+1}^{T} \{Y(t) - \phi(\tau) \ Y(t-\tau)\}^2 \div \sum_{t=\tau+1}^{T} Y^2(t)$$

For each τ, one determines $\phi(\tau)$, and then one determines $\hat{\tau}$ (the optimal value of τ) as the value minimizing

$$\text{Err}(\tau) = \sum_{t=M+1}^{T} \{Y(t) - \phi(\tau) \ Y(t-\tau)\}^2 \div \sum_{t=M+1}^{T} Y^2(t)$$

or

$$\text{Err}(\tau) = \sum_{t=\tau+1}^{T} \{Y(t) - \phi(\tau) \ Y(t-\tau)\}^2 \div \sum_{t=\tau+1}^{T} Y^2(t)$$

The decision as to whether the time series is long memory or not is based on the value of $\text{Err}(\hat{\tau})$. An adhoc rule we use is if $\text{Err}(\hat{\tau}) < 8/T$, the time series is considered long memory. In the present study all time series were judged to be long memory by this criterion. When this criterion fails one often seeks transformations of the form of (2) or (3), using semi-automatic rules described in the appendix.

For the maximum lag M of non-stationary autoregression, the following rules were adopted in this study: M = 2 for yearly series, M = 5 for quarterly series, M = 15 for monthly series.

3. Forecasting Formulas

For forecasting purposes it suffices to adopt for $\tilde{Y}(t)$ a stationary autoregressive model of suitable order m whose coefficients $\alpha_1, \ldots, \alpha_m$ are estimated by Yule Walker equations in the correlation function $\hat{\rho}(v)$ of $\tilde{Y}(t)$. In this paper the model adopted for all time series was of the form

$$\tilde{Y}(t) = Y(t) - \phi(\tau) \ Y(t-\tau)$$
$$\tilde{Y}(t) + \alpha_1 \ \tilde{Y}(t-1) + \ldots + \alpha_m \ \tilde{Y}(t-m) = \epsilon(t)$$

The residual variances are denoted

$$\text{RVY} = \sum_{t=M+1}^{T} \tilde{Y}^2(t) \div \sum_{t=M+1}^{T} Y^2(t)$$

$$\text{RVYT} = \sum_{t=\tau+1}^{T} \epsilon^2(t) \div \sum_{t=\tau+1}^{T} \tilde{Y}^2(t)$$

The last 18 points of the graphs of Y and $\tilde{Y}$ represent not observed values of these series but forecasted values of horizons h = 1 to 18. The mathematical procedure by which they are derived is as follows.

Let

$$Y^{\mu}(t+h|t) = E\{Y(t+h)|Y(t), Y(t-1), \ldots\}$$

denote the predictor of $Y(t+h)$ given values $Y(t)$, $Y(t-1)$, $\ldots$. From the equation

$$Y(t+h) = \phi(\tau) \; Y(t-\tau+h) + \tilde{Y}(t+h)$$

one obtains, by conditioning with respect to $Y(t)$, $Y(t-1)$, $\ldots$

$$Y^{\mu}(t+h|t) = \phi(\tau) \; Y^{\mu}(t-\tau+h|t) + \tilde{Y}^{\mu}(t+h|t)$$

To obtain a formula for forecasts of $\tilde{Y}$ when we have fitted an AR(m) to $\tilde{Y}$:

$$\tilde{Y}(t) + \alpha_1 \; \tilde{Y}(t-1) + \ldots + \alpha_m \; \tilde{Y}(t-m) = \epsilon(t)$$

write

$$\tilde{Y}(t+h) + \alpha_1 \; \tilde{Y}(t+h-1) + \ldots + \alpha_m \; \tilde{Y}(t+h-m) = \epsilon(t+h)$$

$$\tilde{Y}^{\mu}(t+h|t) + \alpha_1 \; \tilde{Y}^{\mu}(t+h-1|t) + \ldots + \alpha_m \; \tilde{Y}^{\mu}(t+h-m|t) = 0$$

One can now compute $Y^{\mu}(t+h/t)$ recursively for $h = 1$, 2, $\ldots$, using the fact that

$$\tilde{Y}^{\mu}(t+j|t) = \tilde{Y}(t+j) \quad \text{if } j \leq 0$$

For example,

$$-\tilde{Y}^{\mu}(t+1|t) = \alpha_1 \; Y(t) + \ldots + \alpha_m \; Y(t-m+1)$$

Then one can compute $Y^{\mu}(t+h/t)$ recursively for $h = 1, 2, \ldots$ using the fact that

$$Y^{\mu}(t+j|t) = Y(t+j) \quad \text{if } j \leq 0$$

For large values of h, one expects $\tilde{Y}^{\mu}(t+h/t) = 0$. Then

$$Y^{\mu}(t+h|t) = \phi(\tau) \; Y^{\mu}(t+h-|t)$$

When $\phi(\tau) \geq 1$, this does not damp down to zero, and provides the long term predictability apparent in many of the series.

4. Summary of Iterated Models Fitted to 111 Time Series

Table I describes the lags of the most significant lag non-stationary scheme for $Y(t)$. For 60% of the monthly series, the annual period ($\hat{\tau} = 12$) was most important; only 26 percent of the quarterly series had an annual period ($\tau = 4$).

The AR character of the residual series $\tilde{Y}(t)$ are described in Table II. Order $m = 0$ indicates white noise (or no memory); 60 % of the yearly series obey the "naive" model $\tilde{Y}(t) = \epsilon(t)$, white noise.

Table III lists the names of 33 series arbitrarily chosen from the set of 111 series to represent typical series. We select this small number of series to discuss in detail. The different types of time series which can be diagnosed by our approach to time

Table I. Lag of Non-stationary AR

τ	1	2	3	4	5	12
20 Yearly	18	2	0	0	0	0
23 Quarterly	15	1	0	6	0	1
68 Monthly	24	0	3	0	0	41

Range of "non-stationary" Coefficients $\phi(\tau)$

	$.99 \leq \phi(\tau) \leq 1.01$	$\phi(\tau) > 1.01$	$\phi(\tau) < .99$
20 Yearly	4	15	1
23 Quarterly	.11	9	3
68 Monthly	27	27	14

Table II. AR Orders Determined by CAT and Innovation Variances

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
20 Yearly	12	4	2	1	0	1	0										
23 Quarterly	8	5	2	1	1	4	2										
68 Monthly	12	18	13	6	3	1	2	1	0	1	1	0	4	2	3	0	1

σ_m^2	.1	.2	.3	.4	.5	.6	.7	.8	.9	.10
20 Yearly	0	0	1	0	1	2	2	1	1	12
23 Quarterly	0	2	2	1	2	4	1	3	0	8
68 Monthly	2	1	2	5	12	9	6	12	7	12

.1 means range $.1 \leq \sigma_m^2 < .2$; similarly for .2 to .9

series modeling and forecasting are illustrated by the results in Table IV and the graphs of Y and $\tilde{Y}$ for the series listed in Table III.

Table IV summarizes the basic model diagnostics of a time series $Y(t)$. These are length; most significant non-stationary autoregressive lagτ, and coefficients $\phi(\tau)$; the residual variance RVY of this non-stationary AR scheme; the best orders (denoted CAT 1 and CAT 2) of approximating AR schemes for $\tilde{Y}(t)$, their horizons HOR 1 and HOR 2, and the residual variance RVYT of the best approximating AR Scheme.

Some ARMA models for quarterly time series were:

OA $\quad \tilde{Y} = (I-1.04L^4)Y, \quad (I-.74L)\tilde{Y} = (I-.85L^4)\varepsilon$

OH $\quad \tilde{Y} = (I-1.02L)Y, \quad (I-.29L^4)\tilde{Y} = (I-.38L^3)\varepsilon$

Some ARMA models for monthly time series were:

MA $\quad \tilde{Y} = (I-1.02L^{12})Y, \quad (I-.41L+.32L^{12})\tilde{Y} = (I-.42L+.31L^5)\varepsilon$

MF $\quad \tilde{Y} = (I-.97L)Y, \quad (I+.31L^{10})\tilde{Y} = (I-.49)\varepsilon$

MJ $\quad \tilde{Y} = (I-1.08L)Y, \quad (I-.75L-.21L^3)\tilde{Y} = (I-.54L^{12})\varepsilon$

MN $\quad \tilde{Y} = (I-1.04L^{12})Y, \quad (I-.29L^2-.28L^3-.27L^{11}+.30L^{13})\tilde{Y} = (I-.42L^{12})\varepsilon$

MR $\quad \tilde{Y} = (I-1.05L^{12})Y, \quad (I-.21L^5-.41L^6)\tilde{Y} = (I-.55L^{12})\varepsilon$

Table III. Typical Series for Detailed Discussion
          (Y,O,M are the prefixes of Yearly,
          Quarterly and Monthly Series Respectively).

YA Machinery and Equipment (YAC 17)
YB National Product and Expenditure-Residential
   Construction (YAC 26)
YC Population Movement Male Death (YAD 6)
YD Crude Birth Rates (YAD 15)
YE Deaths, Analysis by Age and Sex, All Ages,
   United Kingdom (YAD 24)
OA Industrial Production: Textiles (ONI1)
OB Industry Germany (ONI10)
OC Company Data Germany (ONM15)
OD Company Data (ONM6)
OE Industrial Production: Durable Manufactures (QRC13)
OF Industrial Production: Total Austria (QRC22)
OG Value of Manufacturer's New Orders for Consumer Goods (QRC4)
OH Per Capita GNP in Current Dollars (QRG13)
OI Total Industrial Production (QRG4)
MA Company Data (MNB11)
MB Company Data (MNB2)
MC Company Data (MNB20)
MD Company Data (MNB29)
ME Company Data USA (MNB 38)
MF Company Data UK (MNB47)
MG Company Data (MNB56)
MH Company Data (MNB65)
MI Textiles - Quoted at Paris Stock Exchange (MNC17)
MJ General Index of the Industrial Production (MNC26)
MK Reserves - Danemark (MNC35)
ML New Private Housing Units Started Total USA (MNC44)
MM Industrial Production Spain (MNG28)
MN Industrial Production: Finished Investment

Goods Austria (MNG37)
MO Aluminium Production Netherlands (MNI103)
MP Lead Production Canada (MNI122)
MO Production Tin Thailand (MN122)
MR Industrie France(MNI13)
MS Motor Vehicles Production Canada (MNI131)

Table IV. Diagnostics of Model Types of Typical Time Series

Series Length	τ	$\phi(\tau)$	RVY	CAT 1	CAT 2	HOR 1	HOR 2	RVYT
YA 13	1	1.040	.007	2	1	4	2	.66
YB 13	1	1.066	.007	1	2	2	2	1.00
YC 39	1	1.003	.009	5	4	6	6	.58
YD 9	1	.990	.000	1	2	1	1	1.00
YE 13	2	1.010	.001	1	2	1	3	1.00
QA 32	4	1.038	.008	6	3	11	7	.32
QB 57	4	1.023	.003	5	4	5	11	.27
QC 24	1	1.004	.001	1	2	1	2	1.00
QD 20	1	.994	.037	1	7	1	13	1.00
QE 36	1	1.012	.001	1	2	2	2	.86
QF 56	2	1.030	.001	3	4	7	7	.62
QG 52	1	1.011	.009	1	2	1	1	1.00
QH 52	1	1.018	.000	5	3	10	4	.68
QI 56	1	1.013	.000	5	2	14	3	.65

Series Length	τ	φ(τ)	RVY	CAT 1	CAT 2	HOR 1	HOR 2	RVYT
MA 79	12	1.024	.016	2	10	5	24	.66
MB 30	12	1.065	.103	1	2	1	3	1.00
MC 54	1	.973	.057	2	24	4	31	.76
MD 57	1	1.004	.001	2	4	3	5	1.00
ME 32	12	1.261	.050	1	2	2	2	1.00
MF 80	1	.968	.063	9	5	13	5	.65
MG 64	12	1.154	.010	2	10	6	10	.55
MH 52	1	.999	.000	1	3	1	2	1.00
MI 42	1	.993	.002	1	14	2	16	.95
MJ 90	12	1.076	.003	13	19	12	28	.17
MK 5	1	.979	.055	2	7	2	6	.89
ML126	1	1.003	.005	6	2	4	2	.85
MM 43	12	.995	.010	1	2	4	7	.55
MN102	12	1.037	.002	14	3	24	4	.60
MO 65	1	1.006	.003	1	5	2	7	.79
MP 66	1	.949	.100	1	12	2	13	.95
MQ 62	3	.973	.029	6	3	11	7	.57
MR120	12	1.053	.004	16	13	19	19	.63
MS 66	12	.989	.029	3	4	4	5	.81

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Appendix
UNIVARIATE TIME SERIES MODELING AND FORECASTING
AUTOMATIC APPROACHES USING ARARMA MODELS

The model we propose fitting in general to a time series $Y(t)$ is an iterated model (with symbolic transfer functions G and g_∞)

$$Y(t) \longrightarrow \boxed{G} \longrightarrow \tilde{Y}(t) \longrightarrow \boxed{g_\infty} \longrightarrow \varepsilon(t) \text{ white noise}$$

where $\tilde{Y}(t)$ is the results of a "memory shortening" transformation chosen to transform a long memory time series to a short memory one, and g_∞ is an innovation filter which is either an approximating AR filter or an ARMA filter. Parzen (1982) introduces the terminology ARARMA scheme for the iterated time series model with G determined by a non-stationary autoregressive estimation procedure; an ARIMA scheme, introduced by Box and Jenkins (1970), corresponds to a pure differencing operator for G. Autoregressive analysis by Yule-Walker equations yields a stationary autoregressive scheme; a non-stationary autoregressive scheme is one which is fit by estimating its coefficients by ordinary least squares.

To identify the final model, or "overall whitening filter", of a time series, one should determine its model memory type, and identify an iterative model for the time series:

IDENTIFY TIME SERIES MEMORY TYPE		
No Memory (White Noise) (Unpredictable)	Short Memory (Stationary) (Partially Predictable)	Long Memory (Non-stationary) (Predictable)

```
                  │                    │                    │
                  ▼                    ▼                    ▼
                Stop      ┌──────────────────┐  ┌──────────────────┐
                          │     Identify     │  │     Identify     │
                          │                  │  │      Gentle      │
                          │ Whitening Filter │  │ Transformation   │
                          │ as AR(p), MA(q), │  │ to Short Memory  │
                          │  or ARMA(p,q)    │  │  Time Series Ỹ   │
                          └──────────────────┘  └──────────────────┘
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                          ┌──────────────────┐  ┌──────────────────┐
                          │     Estimate     │  │    Model Ỹ by    │
                          │    Parameters    │  │ Whitening Filter │
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                                  │                    │
                                  ▼                    ▼
                             No Memory             No Memory
                             Residuals ε           Residuals ε
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A confirmatory theory of statistical inference is available only for short memory time series (which are ergodic). The modeling of a short memory time series by a whitening filter can be regarded as a science, and it can be made semi-automatic. Given a sample of short memory stationary time series $\tilde{Y}(t)$, our modeling procedure in the time domain is to compute approximating autoregressive schemes.

1. Form the sample correlation function

$$\hat{\rho}(v) = \sum_{t=1}^{T-v} \bar{Y}(t) \ \bar{Y}(t+v) \ \div \ \sum_{t=1}^{T} \bar{Y}^2(t)$$

but do not base any decision upon it, or upon the partial correlations. Rather, compute approximating autoregressive schemes.

2. Solve successive order $m = 1, 2, \ldots$ Yule Walker equations for autoregressive coefficients $\alpha_{1m}, \ldots, \alpha_{mm}$ and residual variance σ_m^2.

3. Use an autoregressive order determining criterion (either CAT or AIC) to determine $\hat{m}(1)$ and $\hat{m}(2)$, the best and second best orders of approximating autoregressive schemes.

4. Compute PVH(h), the prediction variance horizon function for the insight it provides on the memory type and ARMA type of the time series. Compute horizons HOR 1, HOR 2 using approximating AR schemes of orders $\hat{m}(1)$ and $\hat{m}(2)$.

5. Compute a subset AR model.

6. Compute a subset ARMA model.

One can also compute various spectral density functions and spectral distribution functions if one would like the additional insight of the spectral domain.

The diagnosis of a time series as being long memory can be made semi-automatic. Many criteria are available to diagnose time series memory type, using (1) correlations, (2) spectral densities, (3) autoregressive prediction variances, (4) prediction variance horizon function, (5) spectral distribution functions, and (6) S-PLAY diagnostics. The definitions below are given in terms of population parameters, assuming a stationary time series. In practice, the diagnosis is based on sample analogues of these parameters.

The prediction variance horizon PVH(h), $h = 1, 2, \ldots$, is defined in terms of the normalized mean square prediction error of infinite memory prediction h steps ahead:

$$\sigma_{h,\infty}^2 = E\{|Y^{\nu}(t+h|t)|^2\} \div E\{Y^2(t)\}, Y^{\nu}(t+h|t) = Y(t) - Y^{\mu}(t+h|t),$$

$$Y^{\mu}(t+h|t) = E\{Y(t+h)|Y(t), Y(t-1), \ldots\}$$

A formula for $\sigma_{h,\infty}^2$ is obtained by introducing the MA (∞) representation of

$$Y(t) = \epsilon(t) + \beta_1 \epsilon(t-1) + \ldots \ . \quad \text{Then}$$

$$\sigma_{h,\infty}^2 = \sigma_{\infty}^2 \{1 + \beta_1^2 + \ldots + \beta_{h-1}^2\}$$

The graph of $\sigma_{h,\infty}^2$ increases monotonically from σ_{∞}^2 at $h = 1$ to 1 as h tends to ∞. We define

$$PVH(h) = 1 - \sigma_{h,\infty}^2 \quad , \quad h = 1, 2, \ldots$$

and define horizon HOR to be the smallest value of h for which PVH(h) ≤ 0.05 (whence $\sigma_{h,\infty}^2 \geq .95$).

The infinite moving average coefficients β_k are estimated by inverting the transfer function $g_m(z)$ of an approximating autoregressive scheme to obtain, for $k = 1, 2, \ldots$

$$\alpha_0 \beta_k + \alpha_1 \beta_{k-1} + \ldots + \alpha_k \beta_0 = 0$$

The classification of memory type by prediction horizon HOR is:

No Memory HOR = 0	Short Memory 0 < HOR < ∞	Long Memory HOR = ∞

By HOR = ∞, we mean HOR is comparatively large: experiments lead us to conclude that one should compare HOR with the order ORD of the approximating autoregressive scheme. Let HOR/ORD denote the ratio of HOR to ORD; identify time series as follows: If HOR/ORD $\leq$ 1, then MA(q), with q $\leq$ HOR-1. If HOR/ORD $\geq$ 4(say) and PVH decays slowly, then long memory. If PVH declines smoothly and exponentially, then an AR(p) is indicated. If PVH has "bends", then ARMA. If PVH has many level stretches with period τ, then an ARMA model is indicated of the form

$$Y(t) = \frac{I + \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q}{I - \alpha_\tau L^\tau} \varepsilon(t)$$

The final identification of the orders p and q should be by parameter estimation or by use of S-arrays.

The determination of most appropriate "gentle" transformation of Y to $\tilde{Y}$, where Y is long memory and $\tilde{Y}$ is short memory must inevitably involve the physical nature of the observed time series. A semi-automatic approach can be developed by considering the following examples of long memory time series.

A time series Y(t), t = 0, + 1, ..., is called <u>periodic</u> with period τ, if

$$Y(t+\tau) - Y(t) = 0, \text{ all } t.$$

It follows a linear trend Y(t) = a+bt, if for all t

$$Y(t+1) - Y(t) = b, \text{ a constant}$$

It is a pure harmonic of period τ if for all t

$$Y(t) - \phi Y(t-1) + Y(t-2) = 0, \quad \phi = 2 \cos \frac{2\pi}{\tau} .$$

Then

$$Y(t) = A \cos \frac{2\pi}{\tau} t + B \sin \frac{2\pi}{\tau} t .$$

As gentle memory shortening transformations, it is natural to consider

$$\tilde{Y}(t) = Y(t) - \phi(\tau) Y(t-\tau), \qquad\qquad (1)$$

$$\tilde{Y}(t) = Y(t) - \phi_1 Y(t-1) - \phi_2 Y(t-2) \qquad (2)$$
$$\tilde{Y}(t) = Y(t) - \phi_1^1 Y(t-(m-1)) - \phi_2 Y(t-m) \qquad (3)$$

whose coefficients τ, $\phi(\tau)$, ϕ_1, ϕ_2 are determined adaptively from the data. Our first choice is (1); the lag τ is chosen to minimize over τ

$$Err(\tau) = \sum_{t=\tau+1}^{T} \{Y(t) - \phi(\tau)\ Y(t-\tau)\}^2 \div \sum_{t=\tau+1}^{T} Y^2(t)$$

and $\phi(\tau)$ is chosen to minimize over $\phi(\tau)$

$$\sum_{t=\tau+1}^{T} \{Y(t) - \phi(\tau)\ Y(t-\tau)\}^2$$

The stationary correlation function $\rho(\tau)$ of $(Y(t), t = 1, 2, \ldots, T)$ is defined by

$$\beta(\tau) = \sum_{t=1}^{T-\tau} Y(t)\ Y(t+\tau) \div \sum_{t=1}^{T} Y^2(t)$$

Define

$$SSQ(v) = \sum_{t=1}^{v} Y^2(t)$$

One can show that

$$\phi(\tau) = \beta(\tau)\ \frac{SSQ(T)}{SSQ(T-\tau)}$$

$$Err(\tau) = 1 - |\phi(\tau)|^2\ \frac{SSQ(T-\tau)}{SSQ(T)-SSQ(\tau)}$$

The most significant lag $\hat{\tau}$ is defined as the value minimizing Err (τ).

We propose three possible actions at the initial stage of analysis of a time series $(Y(t), t = 1, \ldots, T)$:

L. Declare time series to be long memory, and form $\tilde{Y}(t)$ by (1)

M. Declare time series to be moderately long memory, and form $\tilde{Y}(t)$ by (2).

S. Declare time series to be short memory, and form $\tilde{Y}(t) = Y(t)$, or $\tilde{Y}(t) = Y(t) - \bar{Y}$

where $\bar{Y}$ is the sample mean. After computing $\bar{Y}$, one performs a naive test to decide if it should be set equal to 0; a naive test is $|\bar{Y}| \leq 2\sigma/\sqrt{T}$ where σ is the sample standard deviation.

1. Compute and print $\phi(\tau)$ and Err (τ) for $\tau = 1, 2, \ldots, M$, where M is suitably chosen (15 for yearly, quarterly, or monthly data):

2. Determine $\hat{\tau}$. If Err$(\hat{\tau}) \leq 8/T$, go to L.

3. If $\phi(\hat{\tau}) \geq .9$, and $\hat{\tau} > 2$, go to L.

4. If $\phi(\hat{\tau}) \geq .9$ and $\hat{\tau} = 1$ or 2 determine the best fitting non-stationary AR(2) scheme minimizing.

$$\sum_{t=3}^{T} \{Y(t) - \phi_1 Y(t-1) - \phi_2 Y(t-2)\}^2$$

Let ϕ_1, ϕ_2 denote the minimizing values of ϕ_1 and ϕ_2. Then go to P.

5. If $\phi(\hat{t}) \leq .9$ go to S.

6. If $\phi(\tau)$ is approximately 1 for some τ, one may set this value of τ equal to $\hat{t}$ and go to L. One compares the stationary analysis of this choice of memory shortening transformation with that determined by the value of τ minimizing Err (τ).

7. Non-stationary prediction analysis of a time series in general finds coefficients $\phi_1,\ldots, \phi_m$ minimizing (for a specified memory m)

$$\sum_{t=m+1}^{T} \{Y(t) - \phi_1 Y(t-1) - \ldots - \phi_m Y(t-m)\}^2$$

We recommend a subset regression solution which attempts to determine the most significant lags $j_1, \ldots, j_m$ minimizing

$$\sum_{t=m+1}^{T} \{Y(t) - \phi_{j_1} Y(t-j_1) - \ldots - \phi_{j_n} Y(t-j_n)\}^2$$

and determines the solution for a specified set of lags $j_1, \ldots, j_n$. One may take n = 2, and j_1 and j_2 are two adjacent lags (m-1 and m) for which $\phi(\tau)$ is approximately 1; one then obtains the transformation of type (3).

A model frequently fitted to monthly economic time series is the so-called "airline" model (see Parzen (1979)):

$$(I-L)(I-L^{12}) \, Y(t) = (I-\theta_1 L)(I-\theta_{12}L^{12}) \, \epsilon(t)$$

It seems doubtful that this model would be judged adequate by our criteria, which proposes

$$\tilde{Y}(t) = (I-\phi(12)L^{12}) \, Y(t)$$

$$g_{13}(L) \, \tilde{Y}(t) = \epsilon(t)$$

If one desires a parsimonious ARMA model for $\tilde{Y}(t)$ it may be given by

$$\tilde{Y}(t) + \alpha_1 \tilde{Y}(t-1) + \alpha_{12}\tilde{Y}(t-12) + \alpha_{13}\tilde{Y}(t-13) = \epsilon(t)$$

or

$$\tilde{Y}(t) + \alpha_1 \tilde{Y}(t-1) + \alpha_2 \tilde{Y}(t-2) = \epsilon(t) + \beta_{12}\epsilon(t-12)$$

It should be noted that double differencing is not recommended by us as a memory shortening transformation. When the need for double differencing arises, it appears as a situation in which long memory components continue to be present even after several iterations; then the final iterated model is of the form

$$Y(t) \longrightarrow \tilde{Y}^{(1)}(t) \longrightarrow \tilde{Y}^{(2)}(t) \longrightarrow \epsilon(t)$$

An iterated filter model provides not only forecasts and spectral analysis, but also model interpretation.

Classification of a Time Series into Memory Types

		No Memory	Short Memory	Long Memory
Correlation function	$\bar{\rho} = \sum\limits_{v=1}^{\infty} \lvert \rho(v) \rvert$	$\bar{\rho} = 0$	$0 < \bar{\rho} < \infty$	$\bar{\rho} = \infty$
Spectral density	$M = \dfrac{\max f(\lambda)}{\min f(\lambda)}$	$\log M = 0$	$0 < \log M < \infty$	$\log M = \infty$
Residual variance	σ^2_{∞}	$\sigma^2_{\infty} = 1$	$0 < \sigma^2_{\infty} < 1$	$\sigma^2_{\infty} = 0$
Prediction horizon	HOR	HOR = 0	$0 < HOR < \infty$	$HOR = \infty$
Spectral distribution	$F(\lambda)$	$F(\lambda)$ uniform	$F(\lambda)$ continuous	$F(\lambda)$ has sharp jumps
S-PLAY	S-array of Gray	Infinities in column 1	ARMA(p,q) if Constant row p, Alternating constant column q, Infinity column $-(q+1)$	Constant row 1 (trend) Constant row 2 (seasonal)

Graphs of Y and Ȳ (denoted YT) for the 33 times series listed in
Table III. The break in the graphs indicates the end of the
observed values of the time series and the beginning of predictions
of the next 18 values.
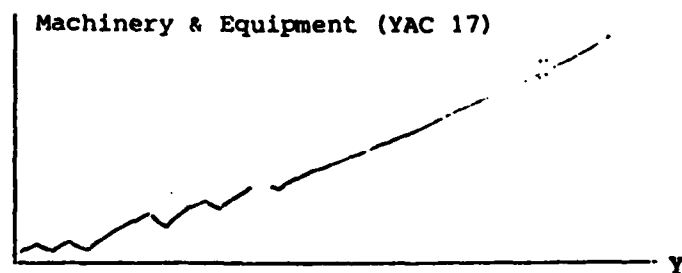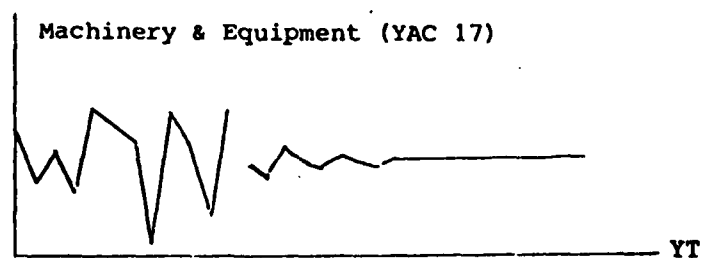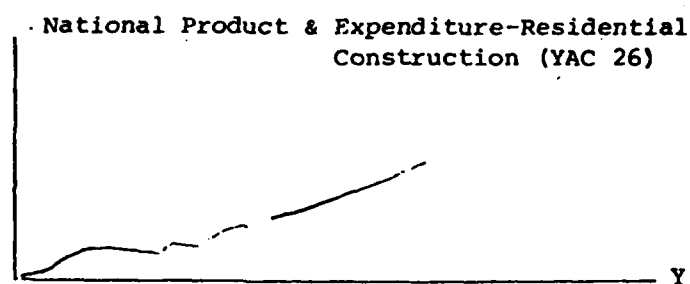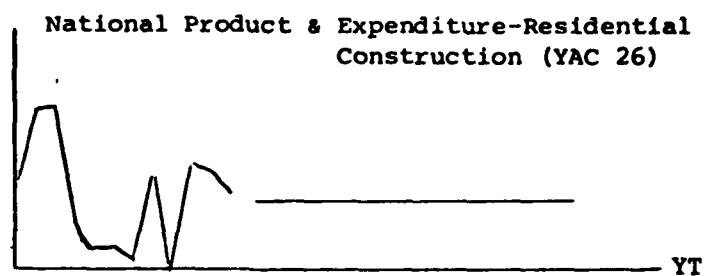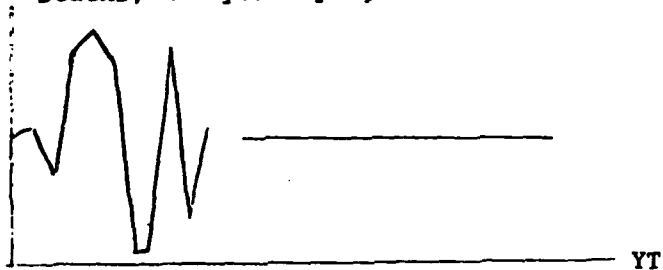
National Product & Expenditure-Residential
Construction (YAC 26)

YT

National Product & Expenditure-Residential
Construction (YAC 26)

Y

Machinery & Equipment (YAC 17)

YT

Machinery & Equipment (YAC 17)

Y

Deaths, Analysis by Age & Sex, All Ages, UK
(YAD 24)

YT

Deaths, Analysis by Age & Sex, All Ages, UK
(YAD 24)

Y

Crude Birth Rates (YAD 15)

YT

Crude Birth Rates (YAD 15)

Y
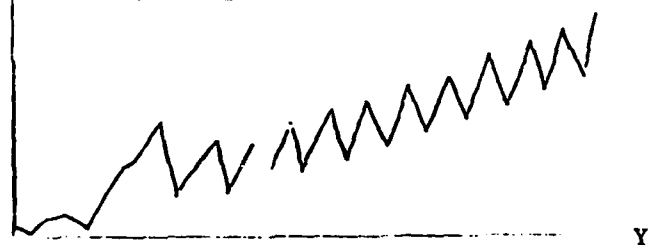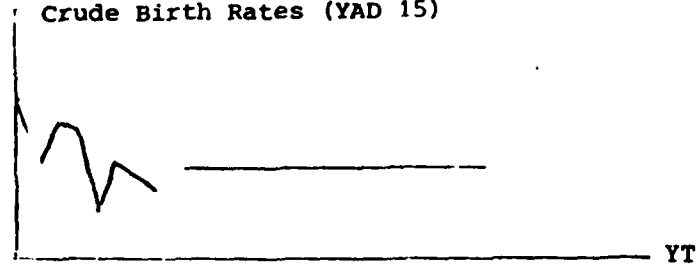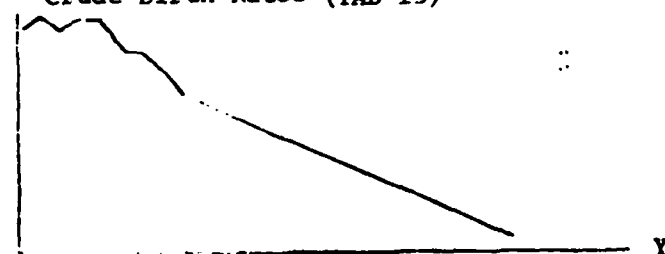
Population Movement Male Death (YAD 6)

Population Movement Male Death (YAD 6)
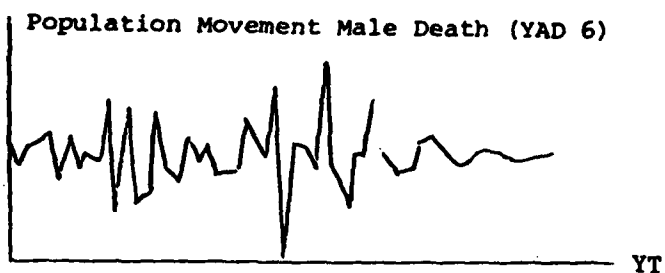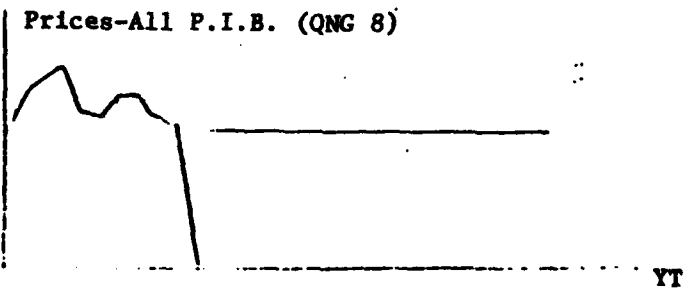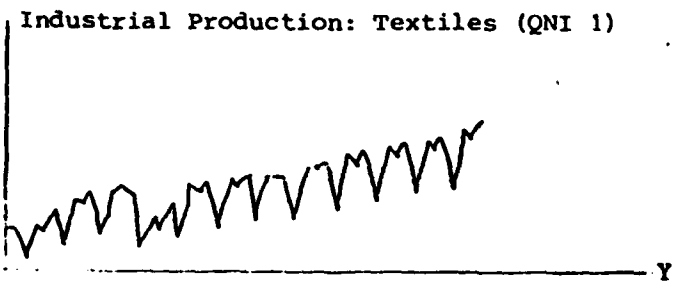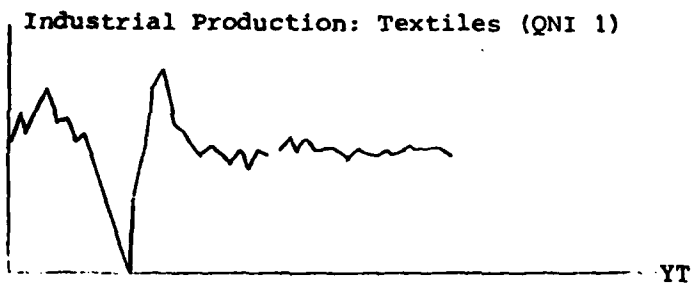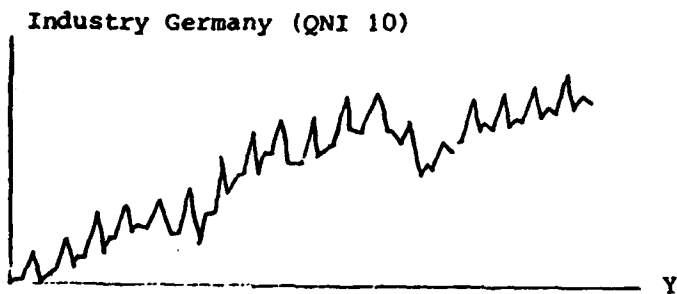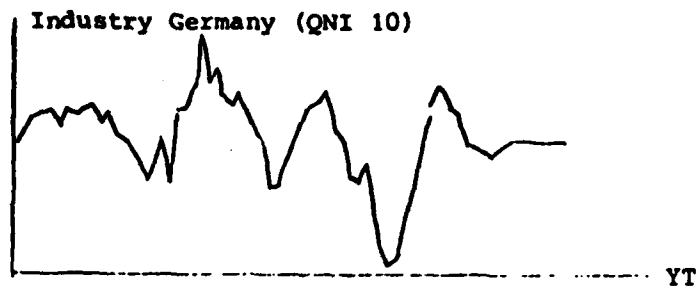
Industry Germany (QNI 10)

YT

Industry Germany (QNI 10)

Y

Industrial Production: Textiles (QNI 1)

YT

Industrial Production: Textiles (QNI 1)

Y

Prices-All P.I.B. (QNG 8)

YT

Value of Manufacturer's New Orders for Consumer
Goods (QRC 4)

YT

Value of Manufacturer's New Orders for Consumer
Goods (QRC 4)

Y

Industrial Production: Total Austria (QRC 22)

YT

Industrial Production: Total Austria (QRC 22)

Y

Industrial Production: Durable Manufactures (QRC 13)

YT
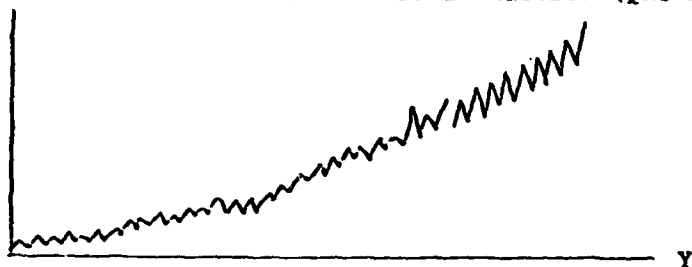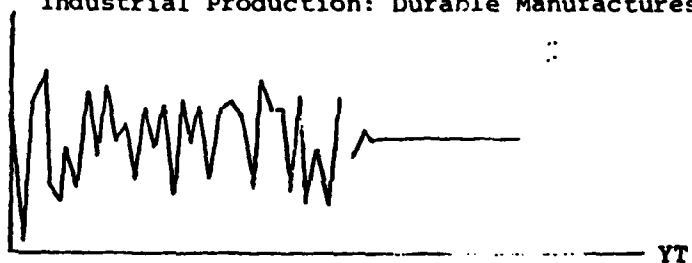
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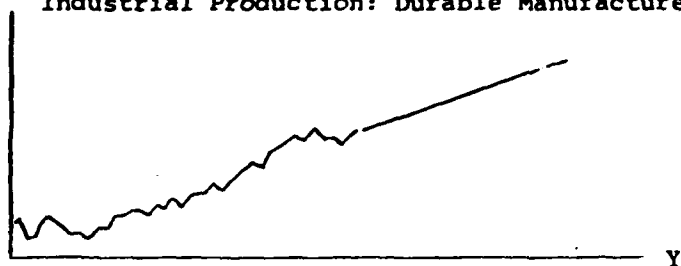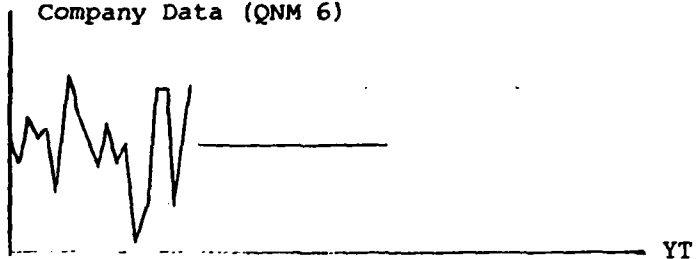
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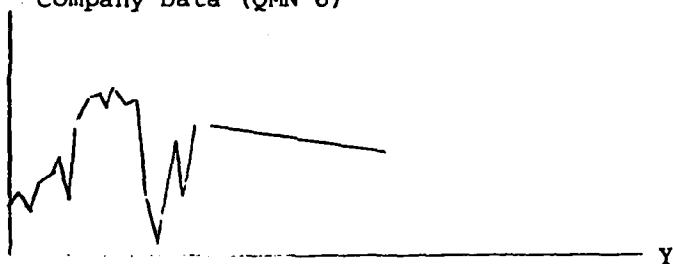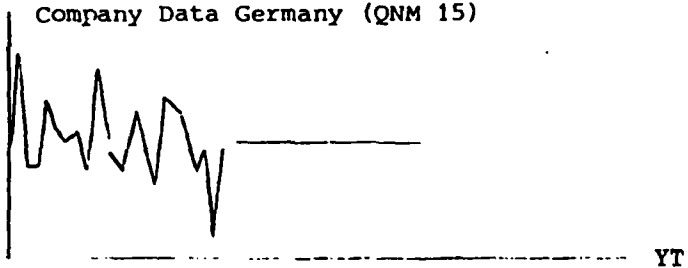
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Company Data Germany (QNM 15)

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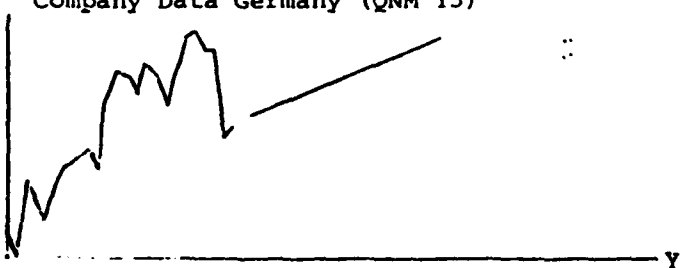
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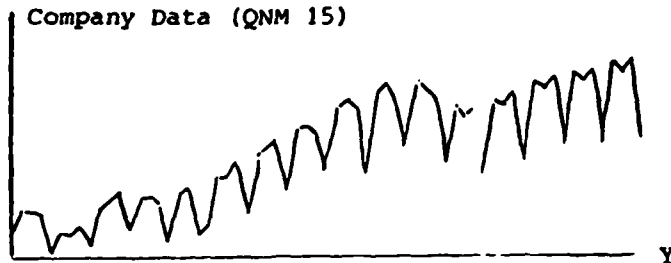
Total Industrial Production (QRG 4)
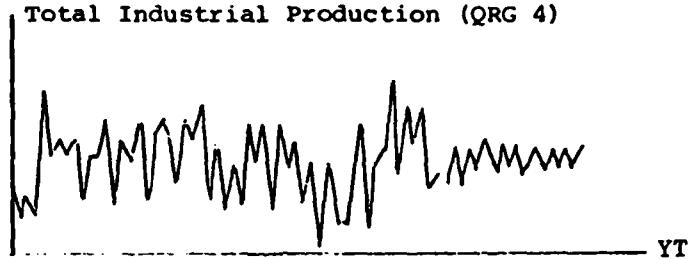
Total Industrial Production (QRG 4)
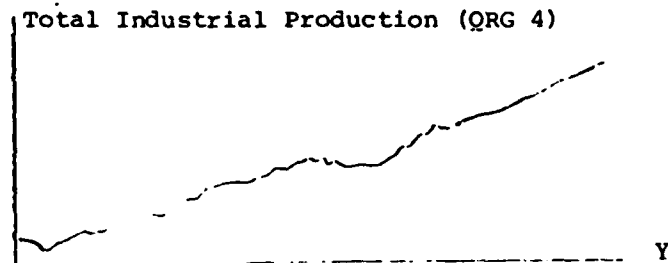
Per Capita GNP In Current Dollars (QRG 13)
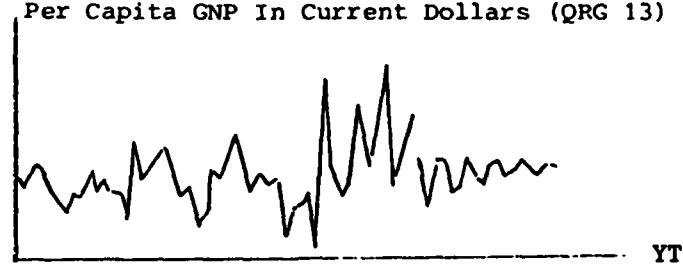
Per Capita GNP In Current Dollars (QRG 13)
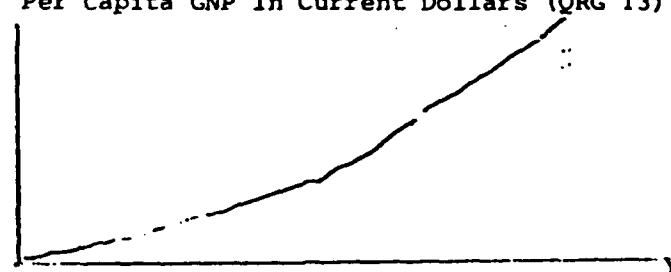
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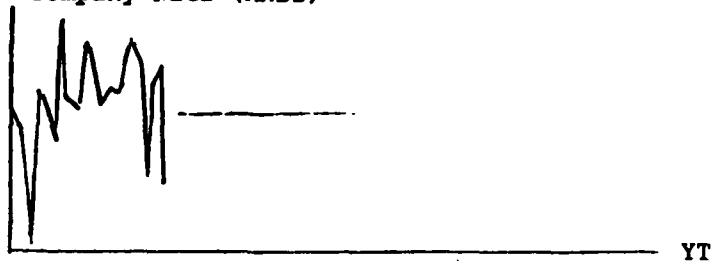
YT

Company Data (MNB2)

Y

Company Data (MNB11)

YT

Company Data (MNB11)

Y

Company Data

YT
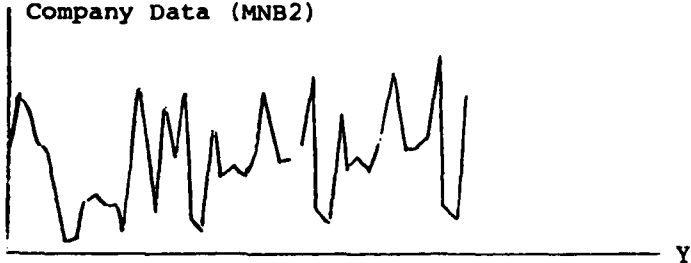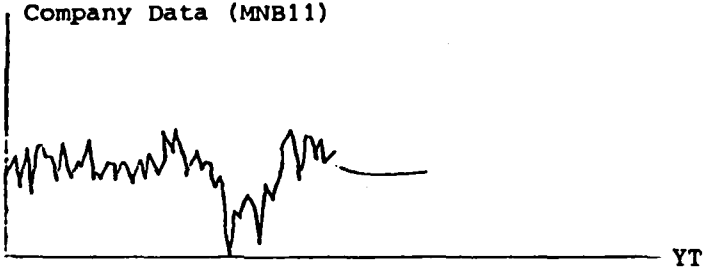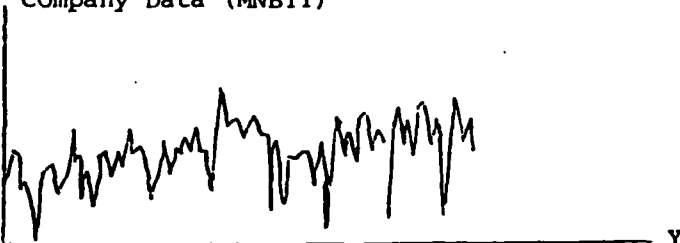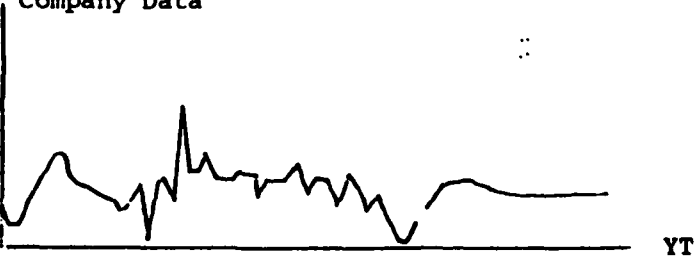
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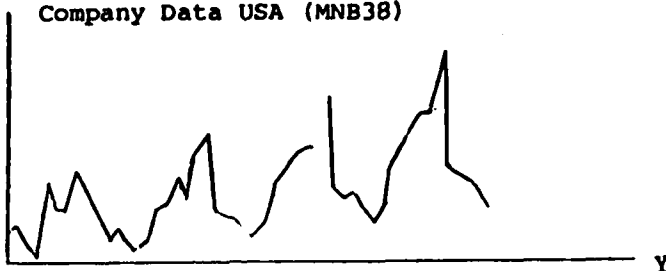
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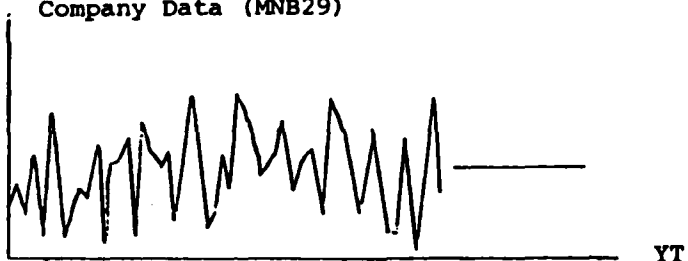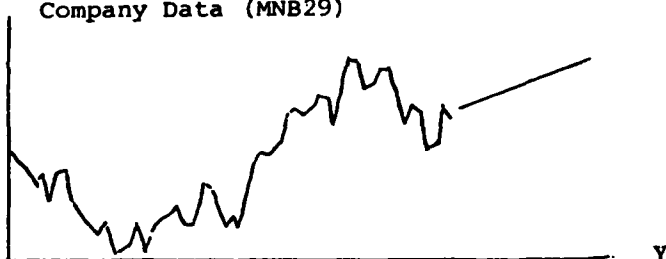
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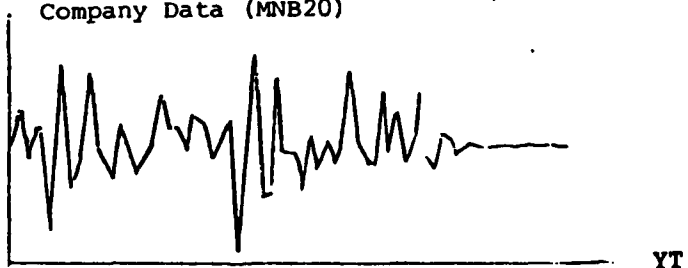
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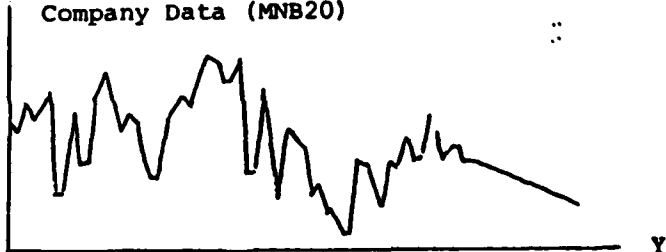
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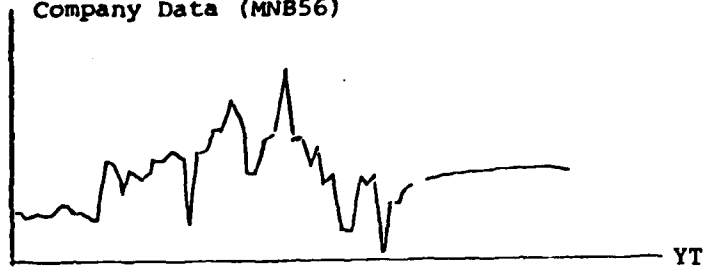
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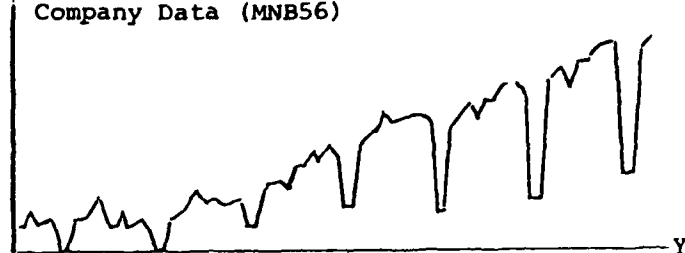
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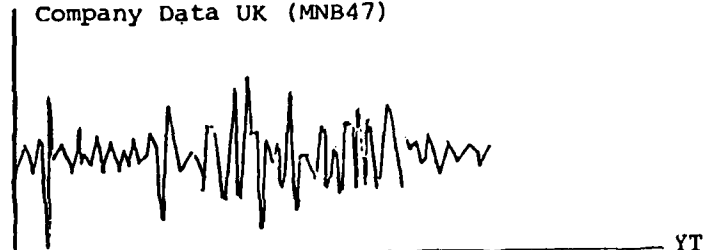
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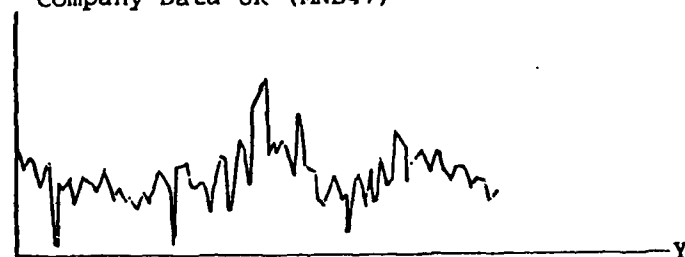
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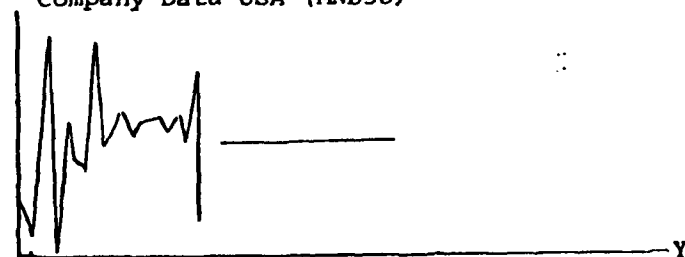
General Index of the Industrial Production (MNC26)
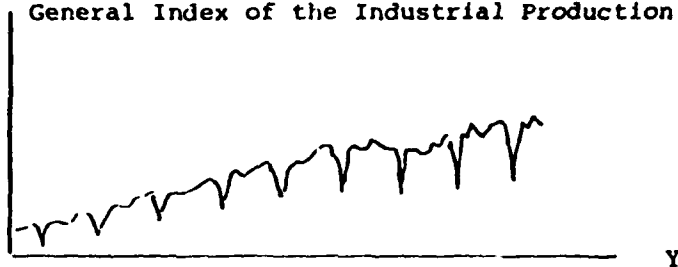
Textiles Quoted at Paris Stock Echange (MNC17)
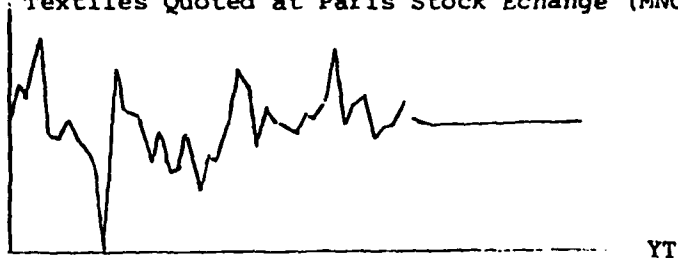
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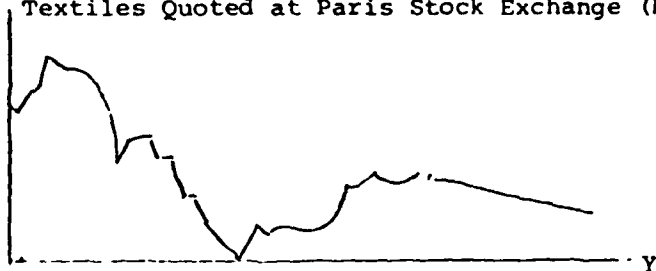
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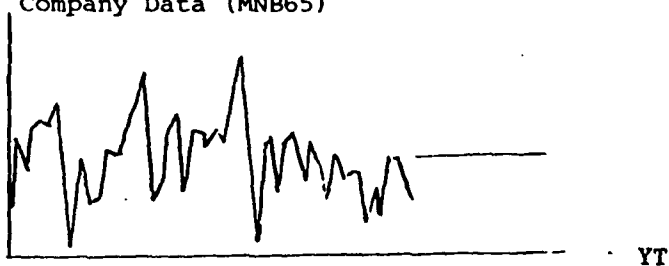
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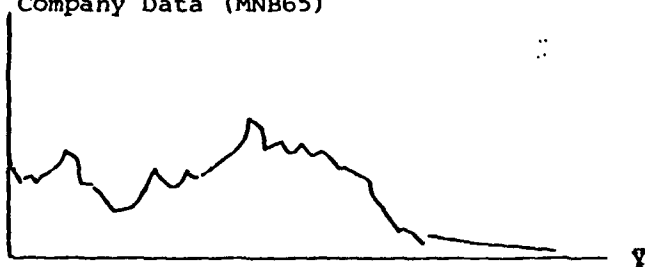
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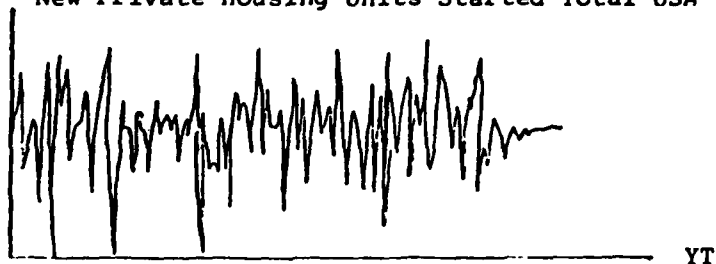
YT

New Private Housing Units Started Total USA (MNC44)

Y

Reserves - Danemark (MNC35)

YT

Reserves - Danemark (MNC35)

Y

General Index of the Industrial Production (MNC26)

YT
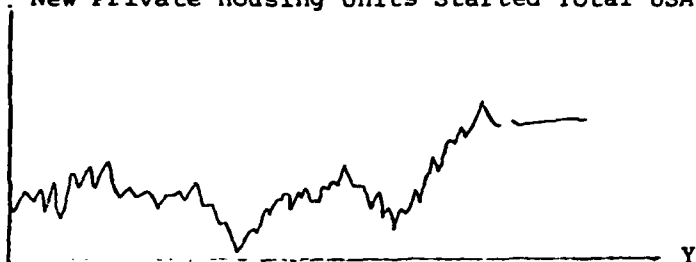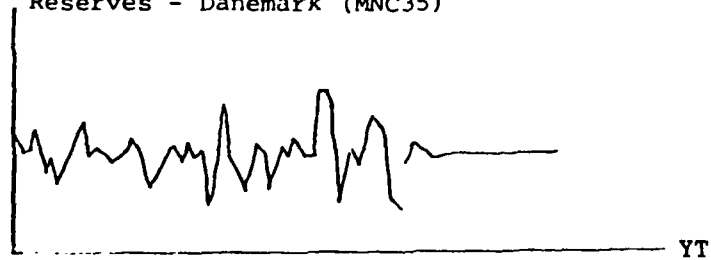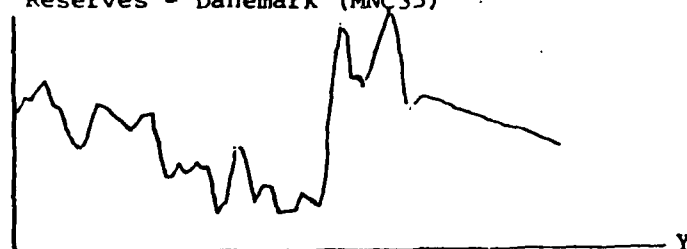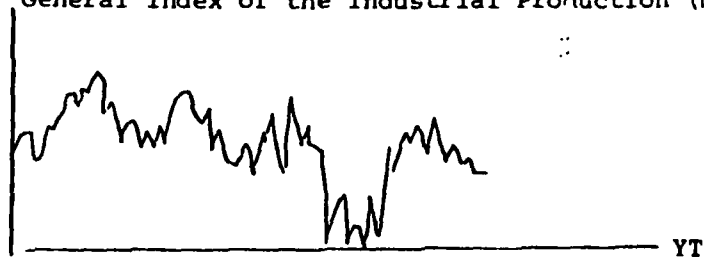
Industrial Production: Finished Investment
Goods Austria (MNG37)

YT
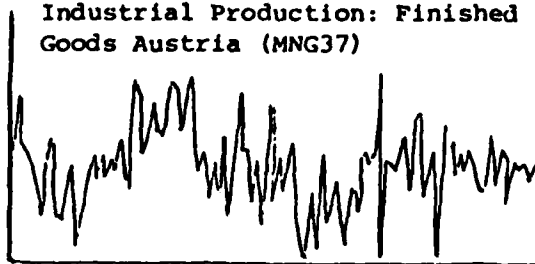
Industrial Production: Finished Investment
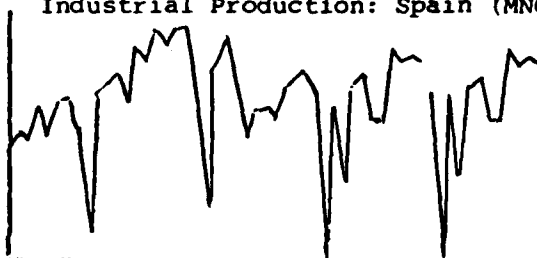Goods Austria (MNG37)
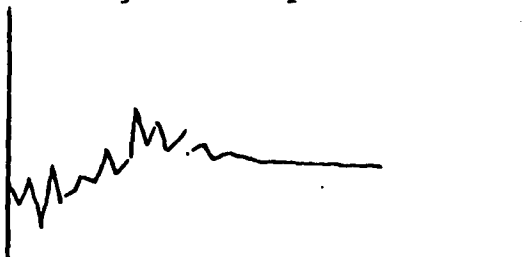

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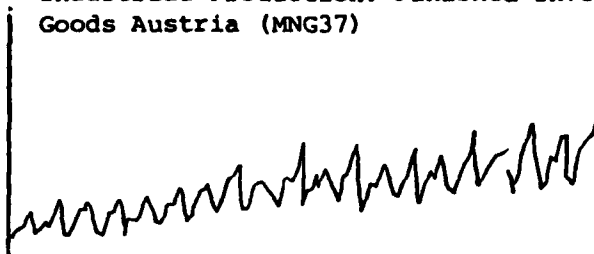
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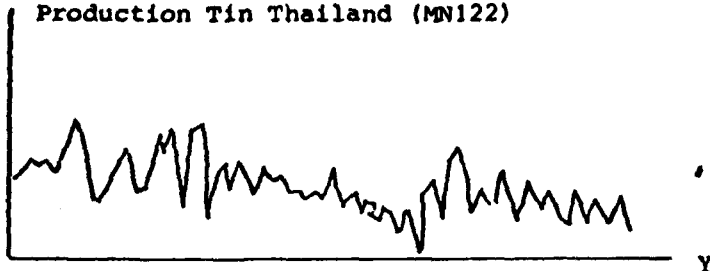

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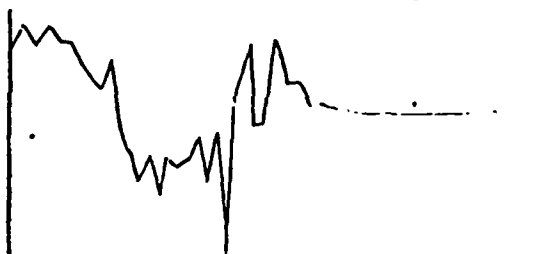
Industrial Production: Spain (MNG28)

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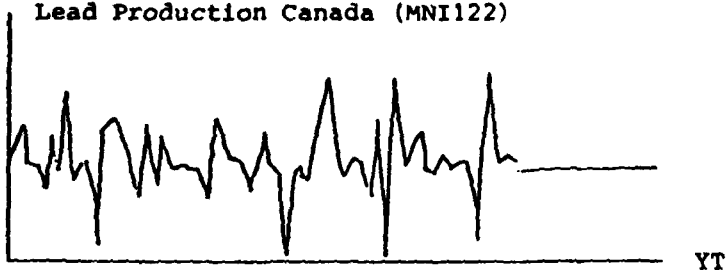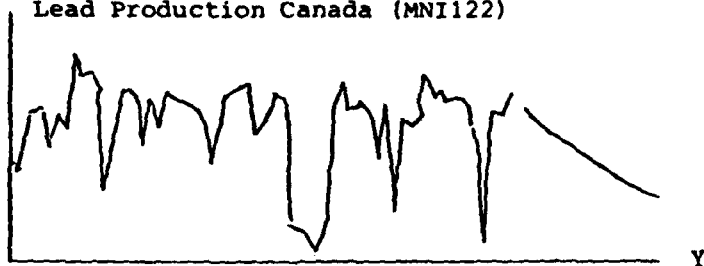Foreign Trade Exports Switzerland
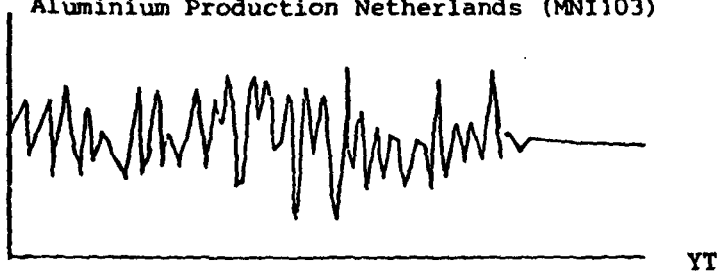

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Production Tin Thailand (MN122)
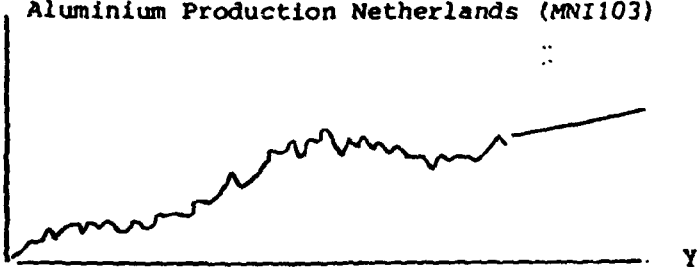

Lead Production Canada (MNI122)

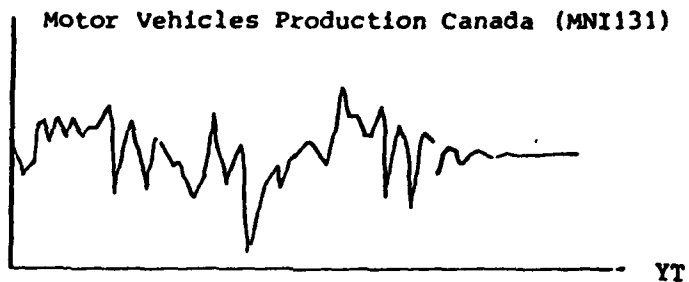**Lead Production Canada (MNI122)**

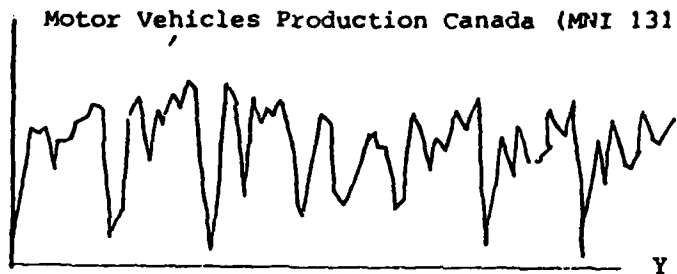**Aluminium Production Netherlands (MNI103)**

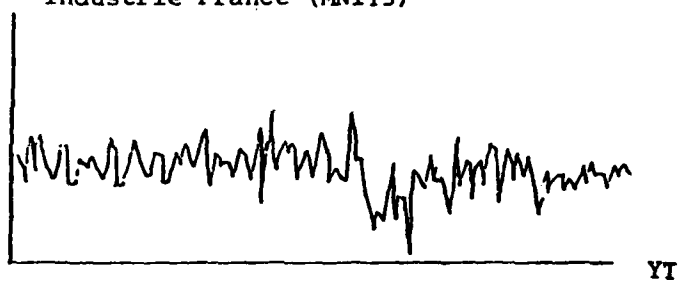**Aluminium Production Netherlands (MNI103)**

Motor Vehicles Production Canada (MNI131)

YT

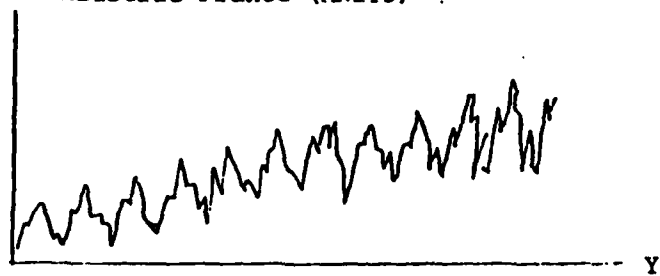**Motor Vehicles Production Canada (MNI 131)**
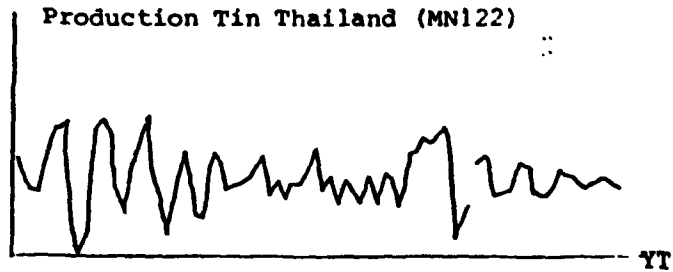
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Industrie France (MNI13)

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Industrie France (MNI13)

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Production Tin Thailand (MN122)

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"HOW TO LEARN FROM THE JOF COMPETITION"

by

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The significance of the "forecasting competition" is best illustrated by comparing it to horse racing. One may distinguish two main types of people at the race track. Type A are bettors; they go to the track to bet on the outcomes of the races and are concerned only with predicting winners. Type B are lovers of knowledge; they go to enjoy the beauty of the horses (and perhaps believe that the purpose of horse-racing is improvement of the breed!), and are satisfied with watching the race.

From a forecasting competition, Type A people want to know who won, which was not explicitly reported in Makridakis et al (1982). The JoF Competition merits publication as a report of raw summaries of the results. Realistically, the authors are not likely to take any action which implies that half of its members are below average. It is appropriate, and desirable, to have subsequent papers that analyze and interpret the results of the forecasting competition. We thank the authors who have provided commentaries in this issue for the enlightenment that they have provided.

Our approach to the forecasting process is based on the belief that a forecasting procedure should, in addition to forecasts, provide knowledge about the "information" in the time series. Important aspects of information are modern versions of the classic idea that a time series can be usefully decomposed into trend, seasonal, and covariance-stationary irregular. Parzen (1981) states that the first step in analysis of a time series is to determine its "memory". "Short memory" corresponds to a covariance-stationary time series for which there are available semi-automatic model identification criteria for fitting AR, MA, and ARMA schemes which transform the "short memory" time series to a "no memory" time series (white noise). "Long memory" contains trend and seasonal components which one seeks to model by regression (on other series or on deterministic functions) or non-stationary autoregression on its past (the first AR in ARARMA).

It is our experience that the transformation of a long memory time series to its "no memory form" has the following "uniqueness" property: if $\epsilon_1(t)$ and $\epsilon_2(t)$ are the white noise residual time series of two different methods of decomposition, then $\epsilon_1(\cdot)$ and $\epsilon_2(\cdot)$ are approximately identically distributed. One usually can conceive of several ways of transforming long memory time

series to a short memory time series; the optimal transformation is not a statistical matter, but depends on how the final overall model is to be applied and interpreted.

Automatic AR and ARMA model identification algorithms can be used to generate analytically several models (called "best" and "second best"), and, thus, forecasts, based on the information contained in past data.

Forecasters should devise systems for comparisons of forecasts generated by different procedures on the time series of interest to their organization, rather than relying on comparisons of other time series. The publication of such case studies should be encouraged.

Our approach to time series analysis is used in the TIMESBOARD library of time series analysis mainline programs and computer subroutines {Newton (1982)}. TIMESBOARD provides tools for a decision-maker seeking forecasting models developed by identifying the information and memory in the time series. Our program DTFORE produces several sets of forecasts for each time series. Each set is optimal in a statistical sense, depending on how the forecaster desires to interpret the diagnostics concerning information and memory of the series. For example, faced with the problem of forecasting a series that is undergoing explosive growth, one can obtain a set of forecasts for continued growth, for leveling off, and for decline. The forecaster, together with the decision maker, can decide which method to use. Of course, the rules of the competition demanded that we produce a single set of forecasts for each series. This was done automatically.

The question remains, then, how to improve the results of the JoF Competition. We have two suggestions.

First, produce plots of the various forecasts appended one above the other, together with the true future values. Obviously, publishing such a graph for 1001 series is impractical. However, a representative sample of each type could be published.

Secondly, forecasting methods are, in our opinion, best compared by forming the time series of forecast errors and studying them. An approach to studying distributions of errors are the quantile and functional statistical inference methods being developed by Parzen (1979) that compute medians, inter-quartile ranges, and various measures of distributional shape.

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